Journal of Ocean Engineering and Science xxx (xxxx) xxx

Contents lists available at ScienceDirect

Journal of Ocean Engineering and Science

journal homepage: www.elsevier.com/locate/joes

The Korteweg-de Vries–Caudrey–Dodd–Gibbon dynamical model: Its conservation laws, solitons, and complexiton

K. Hosseini[a,b,*], A. Akbulut[c], D. Baleanu[d,e,f,*], S. Salahshour[g], M. Mirzazadeh[h,*], K. Dehingia[i]

[a] Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran
[b] Department of Mathematics, Near East University TRNC, Mersin 10, Turkey
[c] Department of Mathematics - Computer, Art-Science Faculty, Eskisehir Osmangazi University, Eskisehir, Turkey
[d] Department of Mathematics, Faculty of Arts and Sciences, Cankaya University, Ankara 06530, Turkey
[e] Institute of Space Sciences, Magurele-Bucharest, Romania
[f] Department of Medical Research, China Medical University, Taichung 40447, Taiwan
[g] Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey
[h] Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157 Rudsar-Vajargah, Iran
[i] Department of Mathematics, Sonari College, Sonari 785690, Assam, India

A R T I C L E I N F O

Article history:
Received 30 April 2022
Revised 19 May 2022
Accepted 1 June 2022
Available online xxx

Keywords:
KdV-CDG dynamical model
Conservation laws
Solitons
Complexiton
Numerical simulations

A B S T R A C T

The main purpose of the present paper is to conduct a detailed and thorough study on the Korteweg-de Vries–Caudrey–Dodd–Gibbon (KdV-CDG) dynamical model. More precisely, after considering the integrable KdV-CDG dynamical model describing certain properties of ocean dynamics, its conservation laws, solitons, and complexiton are respectively derived using the Ibragimov, Kudryashov, and Hirota methods. Several numerical simulations in two and three-dimensional postures are formally given to analyze the effect of nonlinear parameters. It is shown that nonlinear parameters play a key role in the dynamical properties of soliton and complexiton solutions.

1. Introduction

The search for solitons of nonlinear partial differential equations (NLPDEs) plays a fundamental role in a wide variety of nonlinear sciences, as such a class of solutions is capable of giving helpful information regarding the phenomena under investigation. Researchers have devoted much effort to constructing new methods for obtaining solitons of NLPDEs. Some of the methods that have been able to attract the attention of many researchers are the modified Jacobi method [1–4], the exponential method [5–8], and the Kudryashov methods [9–15]. Nowadays, Kudryashov methods, as pioneer approaches, are frequently used to extract solitons of many NLPDEs. Very newly, Hosseini et al. [16] applied successfully Kudryashov methods to derive solitons of a fifth-order nonlinear water wave equation that are classified as W-shaped and bright solitons.

Today, many researchers deal with Lie groups and conservation laws of NLPDEs [17–22] which play a significant role in the solution process of differential equations. As it turns out, researchers face problems in applying Noether's theorem as Euler–Lagrange equations are not available for all differential equations. To overcome this shortcoming, Ibragimov [23] proposed a new conservation theorem that is based on the formal Lagrangian equation, and conservation laws are related to Lie symmetries. Here are some recent papers on the conservation laws of NLPDEs. Arnous et al. [24] obtained conservation laws of the Chen–Lee–Liu equation using the new conservation theorem. Akbulut et al. [25] employed the new conservation theorem to acquire conservation laws of the (3 + 1)-dimensional Wazwaz–KdV equations.

The main purpose of the present paper is to conduct a detailed and thorough study on the following KdV-CDG model [26–31]

$$u_t + c_1\left(u_{xx} + \frac{1}{5}u^2\right)_x + c_2\left(\frac{1}{15}u^3 + uu_{xx} + u_{xxxx}\right)_x = 0, \qquad (1)$$

describing certain properties of ocean dynamics, and obtain its conservation laws, solitons, and complexiton. Eq. (1) as a nonlinear evolutionary equation includes the KdV and CDG equations which have useful applications in nonlinear sciences. Wazwaz [26] utilized Hirota's bilinear method to construct multiple solitons of the KdV-CDG model. Biswas et al. [27] extracted soliton and other solutions of the KdV-CDG model through several effective meth-

* Corresponding authors.
E-mail addresses: kamyar_hosseini@yahoo.com (K. Hosseini), dumitru@cankaya.edu.tr (D. Baleanu), mirzazadehs2@gmail.com (M. Mirzazadeh).

https://doi.org/10.1016/j.joes.2022.06.003

ods. Tu et al. [28] applied Hirota and Riemann methods to derive quasi-periodic and solitary waves of the KdV-CDG model. Akbar et al. [29] found a variety of solitons to the KdV-CDG model using the modified auxiliary equation method. Asjad et al. [30] exerted the hyperbolic function method to derive solitons of the KdV-CDG model. Ma et al. [31] constructed soliton molecules, asymmetric solitons, and hybrid solutions of the KdV-CDG model by considering its N-soliton solutions and applying the velocity resonance method.

The rest of the present paper is as follows: In Section 2, a detailed review of the Ibragimov and Kudryashov methods is given. In Section 3, conservation laws, solitons, and complexiton of the KdV-CDG model are derived. Furthermore, Section 3 presents several numerical simulations in two and three-dimensional postures to analyze the effect of nonlinear parameters in the dynamics of soliton and complexiton solutions. The achievements are reviewed in the last section.

2. Ibragimov and Kudryashov methods: basic ideas

In the current section, the authors are interested in a detailed review of the Ibragimov and Kudryashov methods and their basic ideas.

2.1. Ibragimov method

Conservation theorem: Let us consider

$$F(u, u_x, u_t, \ldots) = 0, \tag{2}$$

as a NLPDE where F is a polynomial.

For Eq. (2), the Lie point symmetry generator is

$$X = \xi^x(x,t,u)\frac{\partial}{\partial x} + \xi^t(x,t,u)\frac{\partial}{\partial t} + \eta(x,t,u)\frac{\partial}{\partial u}, \tag{3}$$

where $\xi^x(x,t,u)$, $\xi^t(x,t,u)$ and $\eta(x,t,u)$ are the infinitesimals. For Eq. (3), the kth prolongation of Eq. (3) is obtained as [32, 33]

$$X^{(k)} = X + \eta_i^{(1)}\frac{\partial}{\partial u_i} + \ldots + \eta^{(k)}_{i_1 i_2 \ldots i_k}\frac{\partial}{\partial u_{i_1 i_2 \ldots i_k}}, k \geq 1,$$

where

$$\eta_i^{(1)} = D_i\eta - \left(D_i\xi^j\right)u_j,$$

$$\eta^{(k)}_{i_1 i_2 \ldots i_k} = D_{i_k}\eta^{(k-1)}_{i_1 i_2 \ldots i_{k-1}} - \left(D_{i_k}\xi^j\right)u_{i_1 i_2 \ldots i_{k-1} j}.$$

The formal Lagrangian is obtained by

$$L = wF,$$

where $w(x,t,u)$ is the adjoint variable. Additionally, the adjoint equation is derived as

$$F^* = \frac{\delta L}{\delta u}, \tag{4}$$

where $\frac{\delta}{\delta u}$ is the variational derivative.

Solving Eq. (4) results in conservation laws of Eq. (2).

Definition: Eq. (2) is said to be nonlinearly self-adjoint if there exists a function [34, 35]

$$w = \phi(x, t, u(x,t)) \neq 0,$$

satisfying

$$F^* = \lambda(x, t, u(x,t))F, \tag{5}$$

where λ is an undetermined coefficient. If we take $w = \phi(u)$ in Eq. (5), Eq. (2) is called quasi self-adjoint. If we take $w = u$, we say that Eq. (2) is strictly self-adjoint.

Theorem 1. *Every Lie point, Lie-Bäcklund, and nonlocal symmetry of Eq. (2) yields a conservation law. The conserved vector components are acquired by [18, 19]*

$$T^i = \xi^i L + W\left[\frac{\partial L}{\partial u_i} - D_j\left(\frac{\partial L}{\partial u_{ij}}\right) + D_j D_k\left(\frac{\partial L}{\partial u_{ijk}}\right) - \cdots\right] + D_j(W)\left[\frac{\partial L}{\partial u_{ij}} - D_k\left(\frac{\partial L}{\partial u_{ijk}}\right) + \cdots\right] + D_j D_k(W)\left[\frac{\partial L}{\partial u_{ijk}}\right] + \cdots, \tag{6}$$

where $W = \eta - \xi^j u_j$. The conserved vectors extracted by Eq. (6) contain the arbitrary solutions of the adjoint equation. Consequently, some conservation laws for Eq. (2) are retrieved by $w(x,t,u)$.

Theorem 2. *Generated conserved vectors using Eq. (6) are conservation laws of Eq. (2) if [17]*

$$D_i\left(T^i\right) = 0.$$

2.2. Kudryashov methods

The KM I applies the following finite series

$$U(\varepsilon) = a_0 + a_1 K(\varepsilon) + a_2 K^2(\varepsilon) + \ldots + a_N K^N(\varepsilon),\ \ a_N \neq 0, \tag{7}$$

as the solution of

$$O\left(U(\varepsilon), U'(\varepsilon), U''(\varepsilon), \ldots\right) = 0. \tag{8}$$

In the above equation, $a_i, i = 0, 1, \ldots, N$ are retrieved later, N is derived by the balance principle, and $K(\varepsilon)$ is of the form

$$K(\varepsilon) = \frac{1}{1 + da^{\varepsilon}},$$

which satisfies

$$K'(\varepsilon) = K(\varepsilon)(K(\varepsilon) - 1)\ln(a).$$

Based on Eqs. (7) and (8), a nonlinear system of algebraic type is obtained, and by solving it, solitons of Eq. (8) are derived.

The series solution of KM II is the same as that considered in KM I. But, KM II benefits from considering

$$K(\varepsilon) = \frac{1}{(A - B)\sinh(\varepsilon) + (A + B)\cosh(\varepsilon)},$$

as the solution of

$$\left(K'(\varepsilon)\right)^2 = K^2(\varepsilon)\left(1 - 4ABK^2(\varepsilon)\right).$$

In a similar way to what was performed before, solitons of Eq. (8) are constructed.

3. KdV-CDG model: its conservation laws, solitons, and complexiton

In the current section, conservation laws, solitons, and complexiton of the KdV-CDG model are derived. Furthermore, several numerical simulations in two and three-dimensional postures are presented to analyze the effect of nonlinear parameters in the dynamics of soliton and complexiton solutions.

3.1. KdV-CDG model and its conservation laws

Based on the conservation theorem, the formal Lagrangian can be written as

$$L = w\left(u_t + c_1\left(u_{xx} + \frac{1}{5}u^2\right)_x + c_2\left(\frac{1}{15}u^3 + uu_{xx} + u_{xxxx}\right)_x\right), \tag{9}$$

where w denotes the adjoint variable.

The adjoint equation is acquired by employing the variational derivative as

$$F^* = -\left(w_t + c_1\left(w_{xxx} + \tfrac{2}{5}w_x u\right) + c_2\left(\tfrac{1}{5}u^2 w_x + 2w_x u_{xx} + 2w_{xx}u_x + uw_{xxx} + w_{xxxxx}\right)\right). \quad (10)$$

If we replace u by w in Eq. (10), then Eq. (1) is not obtained. Thus, the KdV-CDG model is not self-adjoint.

By considering $w = \phi(x, t, u(x, t))$, its derivatives are given by

$$w_t = \phi_u u_t + \phi_t,$$

$$w_x = \phi_u u_x + \phi_x,$$

$$w_{xx} = \phi_u u_{xx} + \phi_{uu}u_x^2 + 2\phi_{ux}u_x + \phi_{xx},$$

$$\vdots$$

Substituting derivatives of ϕ into Eq. (10) without ignoring Eq. (5) yields

$$F^* = -c_1\phi_{xxx} - c_2\phi_{xxxxx} - \frac{2}{5}c_1 u_x u\phi_u - \frac{1}{5}c_2 u^2 u_x\phi_u - 3c_2 uu_x\phi_{xxu} - 3c_2 uu_x^2\phi_{xuuu}$$

$$-c_2 uu_x^3\phi_{uuu} - 4c_2 u_x u_{xx}\phi_u - 3c_2 uu_{xx}\phi_{xu} - c_2 uu_{xxx}\phi_u - \phi_t - 3c_1 u_x\phi_{xxu} - 3c_1 u_x^2\phi_{xuu}$$

$$-c_1 u_x^3\phi_{uuu} - 5c_2 u_x\phi_{xxxxu} - 10c_2 u_x^2\phi_{xxxuu} - 10c_2 u_x^3\phi_{xxuuu} - 5c_2 u_x^4\phi_{xuuuu} - c_2 u_x^5\phi_{uuuu}$$

$$-3c_1 u_{xx}\phi_{xu} - c_1 u_{xxx}\phi_u - 10c_2 u_{xx}\phi_{xxxu} - 10c_2 u_{xxx}\phi_{xxu} - 5c_2 u_{xxxx}\phi_{xu} - c_2 u_{xxxxx}\phi_u$$

$$-15c_2 u_{xx}^2\phi_{xuu} - u_t\phi_u - 3c_2 uu_x u_{xx}\phi_{uu} - 15c_2 u_x u_{xx}^2\phi_{uuu} - 10c_2 u_x^3 u_{xx}\phi_{uuuu}$$

$$-10c_2 u_x^2 u_{xxx}\phi_{uuu} - 10c_2 u_{xx}u_{xxx}\phi_{uu} - 2c_2 u_{xx}\phi_x - 3c_1 u_x u_{xx}\phi_{uu} - 30c_2 u_x u_{xx}\phi_{xxuu}$$

$$-30c_2 u_x^2 u_{xx}\phi_{xuuu} - 20c_2 u_x u_{xxx}\phi_{xuu} - 5c_2 u_x u_{xxxx}\phi_{uu} - \frac{2}{5}c_1 u\phi_x - \frac{1}{5}c_2 u^2\phi_x - 2c_2 u_x\phi_{xx}$$

$$-4c_2 u_x^2\phi_{xu} - 2c_2 u_x^3\phi_{uu} - c_2 u\phi_{xxx} = \lambda\left(u_t + c_1\left(u_{xx} + \frac{1}{5}u^2\right)_x + c_2\left(\frac{1}{15}u^3 + uu_{xx} + u_{xxxx}\right)_x\right).$$

By comparing the coefficients of all derivatives, it is found that

$$\phi_{xu} = 0,$$

$$\phi_x = 0,$$

$$\phi_t = 0,$$

$$\phi_u = 0.$$

As a consequence, one can say that

$$\phi = \sigma_1$$

where σ_1 is a constant. Therefore, $w = 1$ can be considered for obtaining finite conservation laws.

If we apply the fifth-order Lie symmetry generator to Eq. (1), the following infinitesimals are derived:

$$\xi^t = \sigma_1 + \sigma_3 t,$$

$$\xi^x = \sigma_2 + \sigma_3\left(\frac{1}{5}x - \frac{4c_1^2}{25c_2}t\right),$$

$$\eta = -\sigma_3\left(\frac{2c_2 u + 2c_1}{5c_2}\right),$$

where σ_j $(j = 1, 2, 3)$ are constants. As a result, the Lie point symmetry generators admitted by Eq. (1) are obtained as follows

$$X_1 = \tfrac{\partial}{\partial t},\ X_2 = \tfrac{\partial}{\partial x},\ X_3 = \left(\tfrac{1}{5}x - \tfrac{4c_1^2}{25c_2}t\right)\tfrac{\partial}{\partial x} + t\tfrac{\partial}{\partial t} - \left(\tfrac{2c_2 u + 2c_1}{5c_2}\right)\tfrac{\partial}{\partial u}.$$

The commutator table for the acquired symmetries has been given in Table 1.

Table 1
The commutator table for the acquired symmetries.

$[X_i, X_j]$	X_1	X_2	X_3
X_1	0	0	$-\frac{4c_1^2}{25c_2}X_2 + X_1$
X_2	0	0	$\frac{1}{5}X_2$
X_3	$\frac{4c_1^2}{25c_2}X_2 - X_1$	$-\frac{1}{5}X_2$	0

Conservation laws of Eq. (1) can be formulated as follows

$$T^x = \xi^x L + W\left[\tfrac{\partial L}{\partial u_x} - D_x\left(\tfrac{\partial L}{\partial u_{xx}}\right) + D_x^2\left(\tfrac{\partial L}{\partial u_{xxx}}\right) + D_x^4\left(\tfrac{\partial L}{\partial u_{xxxxx}}\right)\right]$$

$$+D_x(W)\left[\tfrac{\partial L}{\partial u_{xx}} - D_x\left(\tfrac{\partial L}{\partial u_{xxx}}\right) - D_x^3\left(\tfrac{\partial L}{\partial u_{xxxxx}}\right)\right] + D_x^2(W)\left[\tfrac{\partial L}{\partial u_{xxx}} + D_x^2\left(\tfrac{\partial L}{\partial u_{xxxxx}}\right)\right]$$

$$+D_x^3(W)\left[-D_x\left(\frac{\partial L}{\partial u_{xxxxx}}\right)\right] + D_x^4(W)\left[\frac{\partial L}{\partial u_{xxxxx}}\right], \quad (11)$$

$$T^t = \xi^t L + W\left[\frac{\partial L}{\partial u_t}\right]. \quad (12)$$

Case 1: From Eqs. (9), (11), and (12) as well as X_1, the following local conservation laws are constructed

$$T_1^x = -w\left(\frac{2}{5}c_1 uu_t + \frac{1}{5}c_2 u^2 u_t + c_2 u_t u_{xx} + c_1 u_{xxt} + c_2 uu_{xxt} + c_2 u_{xxxxt}\right)$$

$$- w_x(c_2 u_t u_x - c_1 u_{xt} - c_2 uu_{xt} - c_2 u_{xxxt}) - w_{xx}(c_1 u_t + c_2 uu_t + c_2 u_{xxt}) - c_2 u_t w_{xxxx} + c_2 u_{xt} w_{xxx},$$

$$T_1^t = w\left(c_1\left(u_{xxx} + \frac{2}{5}uu_x\right) + c_2\left(\frac{1}{5}u^2 u_x + u_x u_{xx} + uu_{xxx} + u_{xxxxx}\right)\right),$$

and we have

$$D_x(T_1^x) + D_t(T_1^t) = u_t w_t - w_t u_t = 0.$$

It is clear that the divergence condition is satisfied.

Assuming $w=1$ leads to

$$\widetilde{T_1^x}=-c_1\left(u_{xxt}+\frac{2}{5}uu_t\right)-c_2\left(\frac{1}{5}u^2u_t+u_tu_{xx}+uu_{xxt}+u_{xxxxt}\right),$$

$$\widetilde{T_1^t}=c_1\left(u_{xxx}+\frac{2}{5}uu_x\right)+c_2\left(\frac{1}{5}u^2u_x+u_xu_{xx}+uu_{xxx}+u_{xxxxx}\right),$$

and $D_x(\widetilde{T_1^x})+D_t(\widetilde{T_1^t})=0$. If we transfer some terms from $\widetilde{T_1^x}$ to $\widetilde{T_1^t}$, zero trivial conservation laws are obtained.

Case 2: From Eqs. (9), (11), and (12) as well as X_2, the following local conservation laws are established

$$\begin{aligned}T_2^x = {}& wu_t-w_x\left(c_2u_x^2-c_1u_{xx}-c_2uu_{xx}-c_2u_{xxxx}\right)\\ &-w_{xx}(c_1u_x+c_2uu_x+c_2u_{xxx})-c_2u_xw_{xxxx}+c_2u_{xx}w_{xxx},\end{aligned}$$

$$T_2^t=-wu_x.$$

It can be demonstrated that the divergence condition is satisfied. By using a similar procedure as above, we find

$$\widetilde{T_2^x}=u_t,$$

$$\widetilde{T_2^t}=-u_x,$$

and so $D_x(\widetilde{T_2^x})+D_t(\widetilde{T_2^t})=0$. If we transfer the term from $\widetilde{T_2^x}$ to $\widetilde{T_2^t}$, zero trivial conservation laws are derived.

Case 3: From Eqs. (9), (11), and (12) as well as X_3, the following local conservation laws are constructed

$$\begin{aligned}T_3^x = {}& \frac{4}{25}c_1^2t(u_xw_{xxxx}-u_{xx}w_{xxx}+u_{xxx}w_{xx}-u_{xxxx}w_x\\ &+c_1\left(u_x^2w_x+uu_xw_{xx}-uu_{xx}w_x\right))\\ &+\frac{4c_1^2}{25c_2}(c_1tu_xw_{xx}-c_1tu_{xx}w_x-uw-wtu_t)\\ &+\frac{1}{5}c_2\left(x\left(\left(uu_xw_{xx}-u_x^2w_x+uu_{xx}w_x\right)+u_{xx}w_{xxx}+u_{xxxx}w_x-u_xw_{xxxx}\right)\right.\\ &-tu^2wu_t-6uwu_{xx}+uu_xw_x-2u^2w_{xx}-2uw_{xxxx}\\ &+3u_xw_{xxx}-6wu_{xxxx}-4u_{xx}w_{xx})\\ &+\frac{1}{5}c_1(xu_{xx}w_x-6wu_{xx}-4uw_{xx}-xu_xw_{xx}-2twuu_t+u_xw_x)\\ &+c_2(u_{xxx}w_x+t(u_{xt}w_{xxx}-u_tw_{xxxx}-u_{xxt}w_{xx}+u_{xxxt}w_x-u_{xxxxt}w\\ &-uu_tw_{xx}+uu_{xt}w_x-uu_{xxt}w-u_tu_xw_x-u_tu_{xx}w))\\ &-\frac{2}{5}c_1w_{xxxx}+\frac{1}{5}xwu_t-\frac{6}{25}c_1wu^2-\frac{2}{25}wc_2u^3\\ &-\frac{2c_1^2}{5c_2}w_{xx}+c_1t(u_{xt}w_x-u_tw_{xx}-u_{xxt}w),\end{aligned}$$

$$\begin{aligned}T_3^t = {}& w\left(c_1t\left(u_{xxx}+\frac{2}{5}uu_x+\frac{4c_1}{25c_2}u_x\right)\right.\\ &\left.+c_2t\left(\frac{1}{5}u^2u_x+u_xu_{xx}+uu_{xxx}+u_{xxxxx}\right)\right)\\ &-\frac{2}{5}uw-\frac{2c_1}{5c_2}w-\frac{1}{5}xwu_x.\end{aligned}$$

A discussion as mentioned in the previous cases regarding the above conservation laws can be stated.

3.2. KdV-CDG model and its solitons

To start, we apply a traveling wave transformation of the form

$$u(x,t)=U(\varepsilon),\ \ \varepsilon=x-\nu t,$$

where ν represents the soliton speed. After employing the above transformation, we find from Eq. (1)

$$\begin{aligned}&-\nu U'(\varepsilon)+c_1\left(U''(\varepsilon)+\tfrac{1}{5}U^2(\varepsilon)\right)'\\ &+c_2\left(\tfrac{1}{15}U^3(\varepsilon)+U(\varepsilon)U''(\varepsilon)+U^{(4)}(\varepsilon)\right)'=0.\end{aligned}\tag{13}$$

By integrating Eq. (13) w.r.t. ε and considering C as the constant of integration, we get

$$\begin{aligned}&-\nu U(\varepsilon)+c_1\left(U''(\varepsilon)+\frac{1}{5}U^2(\varepsilon)\right)\\ &+c_2\left(\frac{1}{15}U^3(\varepsilon)+U(\varepsilon)U''(\varepsilon)+U^{(4)}(\varepsilon)\right)+C=0.\end{aligned}\tag{14}$$

3.2.1. Applying KM I

Based on the linear and nonlinear terms ($U^{(4)}(\varepsilon)$ and $U^3(\varepsilon)$) in Eq. (14), the balance number is acquired as

$N+4=3N\Rightarrow N=2.$

The above balance number and Eq. (7) suggest the following finite series

$$U(\varepsilon)=a_0+a_1K(\varepsilon)+a_2K^2(\varepsilon),\ \ a_2\neq 0,\tag{15}$$

as the solution of Eq. (14). By considering Eqs. (14) and (15) as well as

$$K'(\varepsilon)=K(\varepsilon)(K(\varepsilon)-1)\ln(a),$$

the following system of algebraic type is derived

$$\frac{1}{15}c_2a_0^3+\frac{1}{5}c_1a_0^2-\nu a_0+C=0,$$

$$\begin{aligned}&c_2a_1(\ln(a))^4+a_1(a_0c_2+c_1)(\ln(a))^2\\ &-a_1\left(-\frac{1}{5}a_0^2c_2-\frac{2}{5}a_0c_1+\nu\right)=0,\end{aligned}$$

$$\begin{aligned}&120\left(-c_2\left(\frac{7}{60}a_1-\frac{2}{15}a_2\right)-\frac{1}{120}c_2a_1\right)(\ln(a))^4\\ &+6\left(-\frac{1}{6}a_1(a_0c_2+c_1)+\frac{1}{6}a_1^2c_2-\left(\frac{1}{3}a_1-\frac{2}{3}a_2\right)(a_0c_2+c_1)\right)\\ &(\ln(a))^2+\frac{1}{15}\left(3a_0^2a_2+3a_0a_1^2\right)c_2+\frac{1}{5}a_1^2c_1-\left(-\frac{2}{5}a_0c_1+\nu\right)a_2=0,\end{aligned}$$

$$\begin{aligned}&120\left(-c_2\left(-\frac{3}{10}a_1+\frac{19}{20}a_2\right)+c_2\left(\frac{7}{60}a_1-\frac{2}{15}a_2\right)\right)(\ln(a))^4\\ &+6\left(-\frac{1}{6}a_1^2c_2+\left(\frac{1}{3}a_1-\frac{2}{3}a_2\right)(a_0c_2+c_1)+\frac{1}{6}a_1c_2a_2\right.\\ &\left.-\left(\frac{1}{3}a_1-\frac{2}{3}a_2\right)c_2a_1-a_2(a_0c_2+c_1)\right)\\ &(\ln(a))^2+\frac{1}{15}\left(6a_0a_1a_2+a_1^3\right)c_2+\frac{2}{5}a_1a_2c_1=0,\end{aligned}$$

$$\begin{aligned}&120\left(-c_2\left(\frac{1}{5}a_1-\frac{9}{5}a_2\right)+c_2\left(-\frac{3}{10}a_1+\frac{19}{20}a_2\right)\right)(\ln(a))^4\\ &+6\left(-\frac{7}{6}a_1c_2a_2+\left(\frac{1}{3}a_1-\frac{2}{3}a_2\right)c_2a_1+a_2(a_0c_2+c_1)\right.\\ &\left.-\left(\frac{1}{3}a_1-\frac{2}{3}a_2\right)c_2a_2\right)(\ln(a))^2+\frac{1}{15}\left(3a_0a_2^2+3a_1^2a_2\right)c_2\\ &+\frac{1}{5}a_2^2c_1=0,\end{aligned}$$

$$\begin{aligned}&120\left(-c_2a_2+c_2\left(\frac{1}{5}a_1-\frac{9}{5}a_2\right)\right)(\ln(a))^4\\ &+6\left(\left(\frac{1}{3}a_1-\frac{2}{3}a_2\right)c_2a_2+a_1c_2a_2-c_2a_2^2\right)(\ln(a))^2+\frac{1}{5}c_2a_1a_2^2=0,\end{aligned}$$

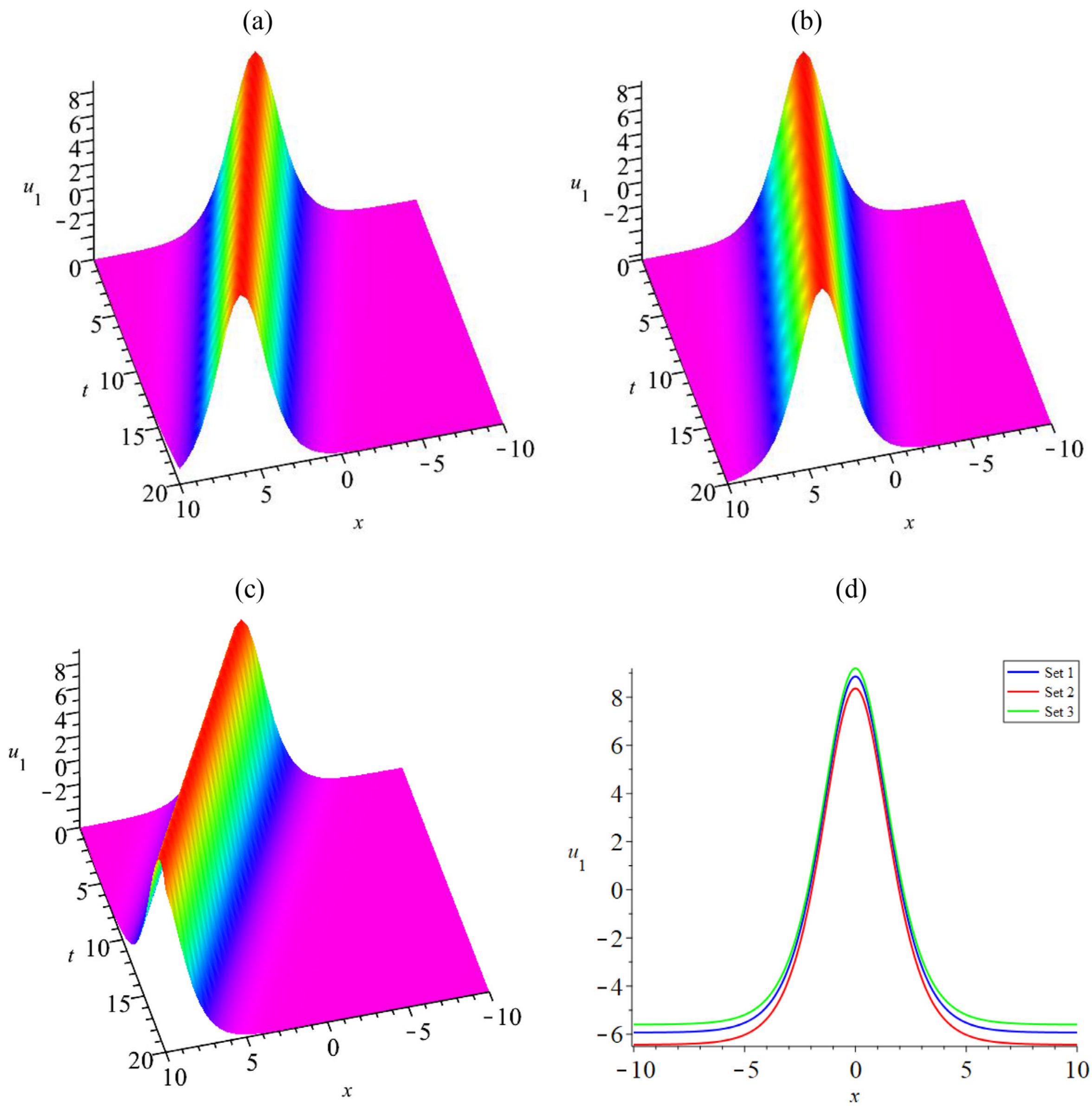

Fig. 1. The first bright soliton for (a) Set 1, (b) Set 2, (c) Set 3, and (d) all sets when $t = 0$.

$120c_2a_2(\ln(a))^4 + 6c_2a_2^2(\ln(a))^2 + \frac{1}{15}c_2a_2^3 = 0.$

Applying a symbolic system like Maple yields the following solutions for the above system:

Case 1:

$$a_0 = -\frac{5(\ln(a))^2c_2 + c_1}{c_2},$$

$$a_1 = 60(\ln(a))^2,$$

$$a_2 = -60(\ln(a))^2,$$

$$v = \frac{5(\ln(a))^4c_2^2 - c_1^2}{5c_2},$$

$$C = \frac{(5(\ln(a))^2c_2+c_1)(10(\ln(a))^4c_2^2-5(\ln(a))^2c_1c_2+c_1^2)}{15c_2^2}.$$

Based on the above results, the following soliton to the KdV-CDG model is derived

$$u_1(x,t) = -\frac{5(\ln(a))^2c_2+c_1}{c_2} + 60(\ln(a))^2\frac{1}{1+da^{x-\frac{5(\ln(a))^4c_2^2-c_1^2}{5c_2}t}} -60(\ln(a))^2\left(\frac{1}{1+da^{x-\frac{5(\ln(a))^4c_2^2-c_1^2}{5c_2}t}}\right)^2.$$

Case 2:

$$a_1 = 30(\ln(a))^2,$$

$$a_2=-30(\ln(a))^2,$$

$$\nu=(\ln(a))^4c_2+(\ln(a))^2a_0c_2+(\ln(a))^2c_1+\frac{1}{5}a_0^2c_2+\frac{2}{5}a_0c_1,$$

$$C=\frac{2}{15}c_2a_0^3+\frac{1}{5}c_1a_0^2+a_0(\ln(a))^4c_2+(\ln(a))^2a_0^2c_2+a_0(\ln(a))^2c_1.$$

Based on the above results, the following soliton to the KdV-CDG model is obtained

$$u_2(x,t)=a_0+30(\ln(a))^2\frac{1}{1+da^{x-\left((\ln(a))^4c_2+(\ln(a))^2a_0c_2+(\ln(a))^2c_1+\frac{1}{5}a_0^2c_2+\frac{2}{5}a_0c_1\right)t}}$$
$$-30(\ln(a))^2\left(\frac{1}{1+da^{x-\left((\ln(a))^4c_2+(\ln(a))^2a_0c_2+(\ln(a))^2c_1+\frac{1}{5}a_0^2c_2+\frac{2}{5}a_0c_1\right)t}}\right)^2$$

To analyze the effect of nonlinear parameters (c_1 and c_2) in the dynamics of the first bright soliton, several numerical simulations in two and three-dimensional postures are formally given. The following families

Set 1: $\{c_1=0.4, c_2=0.4, a=2.7, d=1\}$,

Set 2: $\{c_1=0.6, c_2=0.4, a=2.7, d=1\}$,

Set 3: $\{c_1=0.4, c_2=0.6, a=2.7, d=1\}$, have been taken to plot Fig. 1.

3.2.2. Applying KM II

By considering Eqs. (14) and (15) as well as

$$\left(K'(\varepsilon)\right)^2=K^2(\varepsilon)\left(1-4ABK^2(\varepsilon)\right),$$

the following nonlinear algebraic system is acquired

$$\left(AB-\frac{1}{120}a_2\right)\left(AB-\frac{1}{240}a_2\right)=0,$$

$$A^2B^2-\frac{1}{12}ABa_2+\frac{1}{1920}a_2^2=0,$$

$$\frac{1}{15}c_2\left((-120AB+3a_2)a_1^2-360\left(AB-\tfrac{1}{120}a_2\right)a_2(a_0+20)\right)$$
$$-24\left(AB-\tfrac{1}{120}a_2\right)c_1a_2=0,$$

$$\left(-\tfrac{1}{120}a_1^2+\left(-\tfrac{1}{20}a_0-\tfrac{5}{8}\right)a_2+AB(a_0+10)\right)c_2$$
$$+c_1\left(AB-\tfrac{1}{20}a_2\right)=0,$$

$$\tfrac{1}{15}\left((3a_0+15)a_1^2+3a_2\left(a_0^2+20a_0+80\right)\right)c_2$$
$$+\tfrac{1}{5}a_1^2c_1-a_2\left(-\tfrac{2}{5}a_0c_1+\nu-4c_1\right)=0,$$

$$\left(a_0^2+5a_0+5\right)c_2+2a_0c_1-5\nu+5c_1=0,$$

$$\frac{1}{15}c_2a_0^3+\frac{1}{5}c_1a_0^2-\nu a_0+C=0.$$

Employing a symbolic system like Maple results in the following solutions for the above system:

Case 1:

$$a_1=0,$$

$$a_2=120AB,$$

$$\nu=\frac{1}{5}a_0^2c_2+\frac{2}{5}a_0c_1+4a_0c_2+4c_1+16c_2,$$

$$C=\tfrac{2}{15}c_2a_0^3+\tfrac{1}{5}c_1a_0^2+4a_0^2c_2+4a_0c_1+16a_0c_2.$$

Based on the above results, the following soliton to the KdV-CDG model is derived

$$u_3(x,t)$$
$$=a_0+120AB\left(\frac{1}{(A-B)\sinh\left(x-\left(\frac{1}{5}a_0^2c_2+\frac{2}{5}a_0c_1+4a_0c_2+4c_1+16c_2\right)t\right)+(A+B)\cosh\left(x-\left(\frac{1}{5}a_0^2c_2+\frac{2}{5}a_0c_1+4a_0c_2+4c_1+16c_2\right)t\right)}\right)^2.$$

Case 2:

$$a_0=-\frac{c_1+20c_2}{c_2},$$

$$a_1=0,$$

$$a_2=240AB,$$

$$\nu=-\frac{c_1^2-80c_2^2}{5c_2},$$

$$C=\frac{(c_1+20c_2)(c_1^2-20c_1c_2+160c_2^2)}{15c_2^2}.$$

Based on the above results, the following soliton to the KdV-CDG model is derived

$$u_4(x,t)=-\frac{c_1+20c_2}{c_2}$$
$$+240AB\left(\frac{1}{(A-B)\sinh\left(x+\left(\frac{c_1^2-80c_2^2}{5c_2}\right)t\right)+(A+B)\cosh\left(x+\left(\frac{c_1^2-80c_2^2}{5c_2}\right)t\right)}\right)^2.$$

Several two and three-dimensional representations are formally given to investigate the effect of nonlinear parameters (c_1 and c_2) in the dynamics of the fourth bright soliton. The following groups

Set 1: $\{A=1, B=2, c_1=0.01, c_2=0.01\}$,

Set 2: $\{A=1, B=2, c_1=0.04, c_2=0.01\}$,

Set 3: $\{A=1, B=2, c_1=0.01, c_2=0.06\}$, have been adopted to portray Fig. 2.

3.3. KdV-CDG model and its complexiton

To extract the complexiton of the KdV-CDG model, the following assumptions are considered [36, 37]

$$\mu=\mu_1+i\mu_2,$$

$$\nu=\nu_1+i\nu_2,$$

$$p(x,t)=xt+c_1x^4+c_2x^6.$$

From

$$p(\mu,\nu)=0,$$

$$p(\overline{\mu},\overline{\nu})=0,$$

and exerting a few operations, a nonlinear algebraic system is acquired as follows

$$6c_2\mu_1^5\mu_2+4\left(-5c_2\mu_2^2+c_1\right)\mu_2\mu_1^3+\left(6c_2\mu_2^5-4c_1\mu_2^3+\nu_2\right)\mu_1$$
$$+\mu_2\nu_1=0,$$

$$c_2\mu_1^6+\left(-15c_2\mu_2^2+c_1\right)\mu_1^4+\left(15c_2\mu_2^4-6c_1\mu_2^2\right)\mu_1^2+\mu_1\nu_1$$
$$+\left(-c_2\mu_2^5+c_1\mu_2^3-\nu_2\right)\mu_2=0.$$

Applying a symbolic system like Maple yields

$$\nu_1=-\mu_1\left(c_2\mu_1^4-10c_2\mu_1^2\mu_2^2+5c_2\mu_2^4+c_1\mu_1^2-3c_1\mu_2^2\right),$$

$$\nu_2=-5c_2\mu_1^4\mu_2+10c_2\mu_1^2\mu_2^3-c_2\mu_2^5-3c_1\mu_1^2\mu_2+c_1\mu_2^3.$$

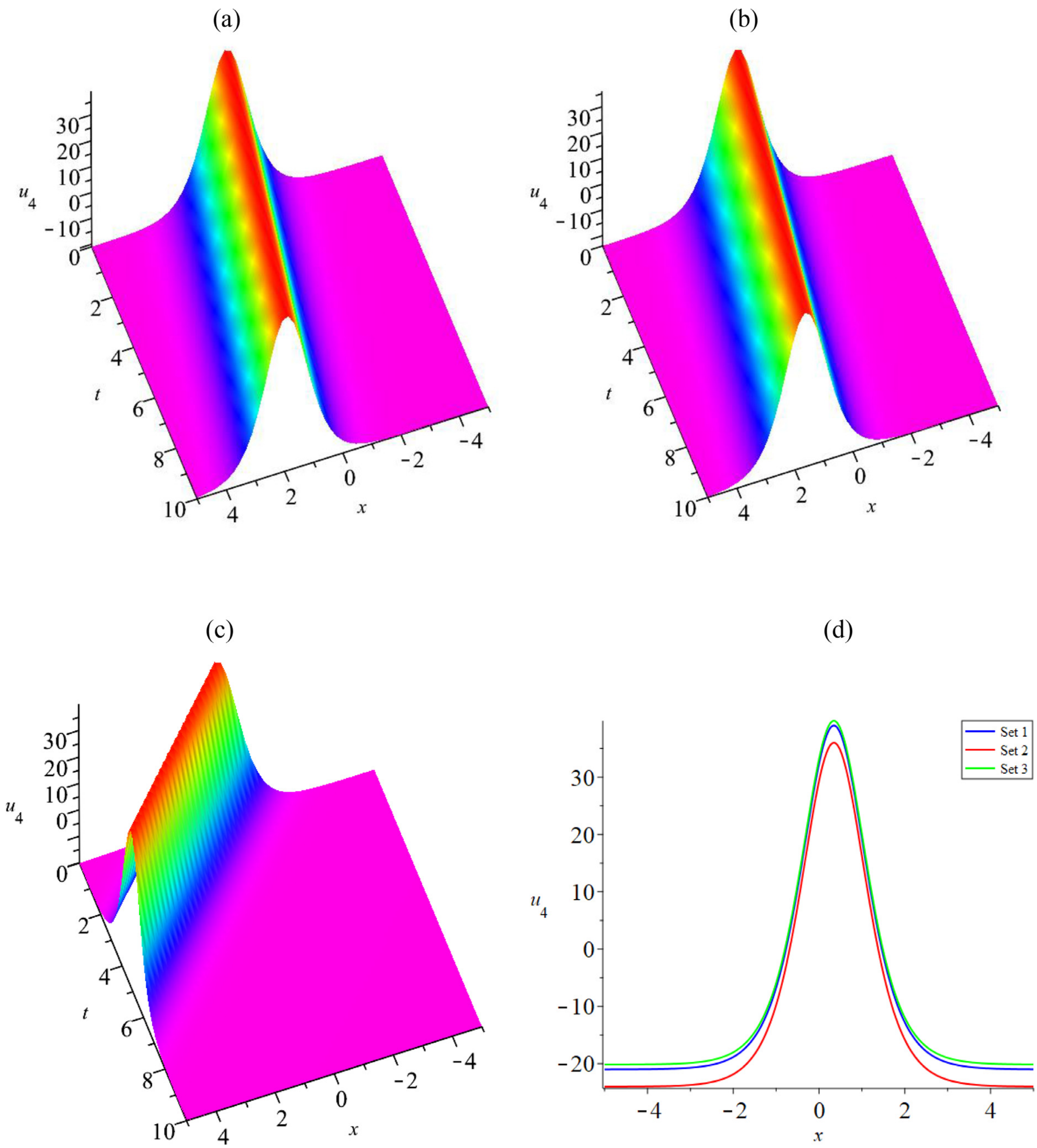

Fig. 2. The fourth bright soliton for (a) Set 1, (b) Set 2, (c) Set 3, and (d) all sets when $t = 0$.

Through the command

$$a_{12} = -\frac{p(2i\mu_2, 2i\nu_2)}{p(2\mu_1, 2\nu_1)},$$

the phase shift is found as

$$a_{12} = -\frac{-64c_2\mu_2^6+16c_1\mu_2^4-4\mu_2\left(-5c_2\mu_1^4\mu_2+10c_2\mu_1^2\mu_2^3-c_2\mu_2^5-3c_1\mu_1^2\mu_2+c_1\mu_2^3\right)}{64c_2\mu_1^6+16c_1\mu_1^4-4\mu_1^2\left(c_2\mu_1^4-10c_2\mu_1^2\mu_2^2+5c_2\mu_2^4+c_1\mu_1^2-3c_1\mu_2^2\right)}.$$

Now, the following complexiton to the KdV-CDG model is derived

$$u_5(x,t) = 30(\ln(f(x,t)))_{xx},$$

where

$$f(x,t) = 1 + 2e^{\vartheta_1}\cos(\vartheta_2) + a_{12}e^{2\vartheta_1},$$

$$\vartheta_i = \mu_i x + \nu_i t, \ \ i = 1, 2,$$

$$\nu_1 = -\mu_1\left(c_2\mu_1^4 - 10c_2\mu_1^2\mu_2^2 + 5c_2\mu_2^4 + c_1\mu_1^2 - 3c_1\mu_2^2\right),$$

$$\nu_2 = -5c_2\mu_1^4\mu_2 + 10c_2\mu_1^2\mu_2^3 - c_2\mu_2^5 - 3c_1\mu_1^2\mu_2 + c_1\mu_2^3,$$

$$a_{12} = -\frac{-64c_2\mu_2^6+16c_1\mu_2^4-4\mu_2(-5c_2\mu_1^4\mu_2+10c_2\mu_1^2\mu_2^3-c_2\mu_2^5-3c_1\mu_1^2\mu_2+c_1\mu_2^3)}{64c_2\mu_1^6+16c_1\mu_1^4-4\mu_1^2(c_2\mu_1^4-10c_2\mu_1^2\mu_2^2+5c_2\mu_2^4+c_1\mu_1^2-3c_1\mu_2^2)}.$$

To analyze the effect of nonlinear parameters (c_1 and c_2) in the dynamics of the complexiton, several numerical simulations in three-dimensional postures are formally given. The following families

Set 1: $\{\mu_1 = 0.5, \mu_2 = 0.5, c_1 = 0.01, c_2 = 1\}$,
Set 2: $\{\mu_1 = 0.5, \mu_2 = 0.5, c_1 = 0.06, c_2 = 1\}$,

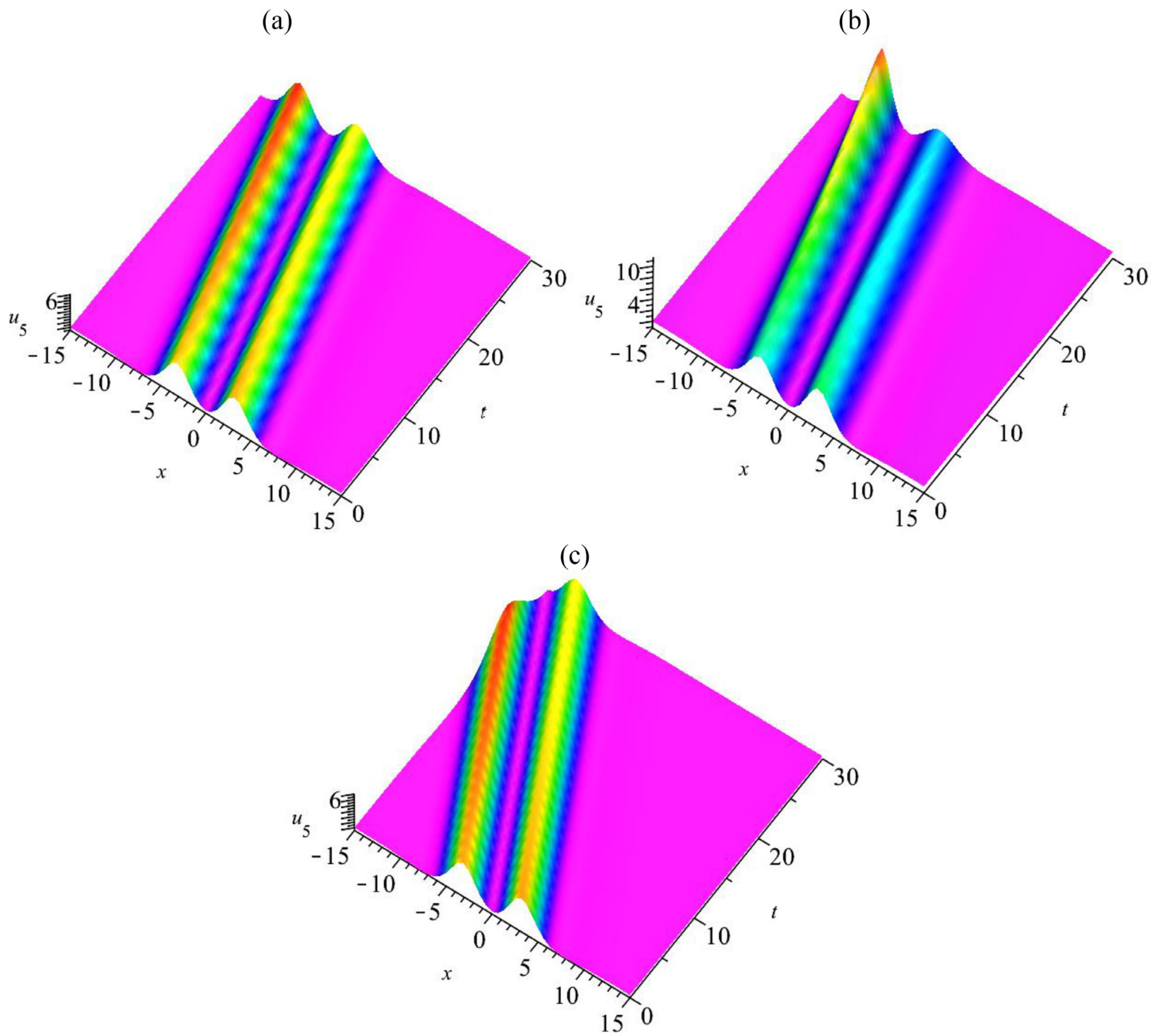

Fig. 3. The complexiton for (a) Set 1, (b) Set 2, and (c) Set 3.

Set 3: $\{\mu_1 = 0.5, \mu_2 = 0.5, c_1 = 0.01, c_2 = 2\}$, have been taken to plot Fig. 3.

4. Conclusion

In the current paper, the authors conducted a new and complete investigation on the Korteweg-de Vries–Caudrey–Dodd–Gibbon dynamical model describing certain properties of ocean dynamics. First, by adopting the Ibragimov method which is based on the formal Lagrangian equation, the local conservation laws of the KdV-CDG model were formally derived. Kudryashov and Hirota methods were then applied to the KdV-CDG model to derive its solitons and complexiton. Several numerical simulations in two and three-dimensional postures were formally presented to examine the effect of nonlinear parameters in the dynamics of soliton and complexiton solutions. It was observed that the change of nonlinear parameters has a significant effect on the dynamical evolution of solitons and complexiton. As future works, the authors' concern is to adopt other well-designed methods [38–50] to construct other wave structures of the KdV-CDG model.

Declaration of Competing Interest

No conflict of interest exists in the submission of this manuscript, and the manuscript is approved by all authors for publication.

The authors declare that they have no competing interests.

References

[1] H.C. Ma, Z.P. Zhang, A.P. Deng, A new periodic solution to Jacobi elliptic functions of MKdV equation and BBM equation, Acta Math. Appl. Sin. 28 (2012) 409–415.
[2] M.M.A. El-Sheikh, A.R. Seadawy, H.M. Ahmed, A.H. Arnous, W.B. Rabie, Dispersive and propagation of shallow water waves as a higher order nonlinear Boussinesq-like dynamical wave equations, Phys. A 537 (2020) 122662.
[3] K. Hosseini, S. Salahshour, M. Mirzazadeh, A. Ahmadian, D. Baleanu, A. Khoshrang, The (2+1)-dimensional Heisenberg ferromagnetic spin chain equation: its solitons and Jacobi elliptic function solutions, Eur. Phys. J. Plus 136 (2021) 206.
[4] K. Hosseini, K. Sadri, M. Mirzazadeh, S. Salahshour, An integrable (2+1)-dimensional nonlinear Schrödinger system and its optical soliton solutions, Optik 229 (2021) 166247.
[5] J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations, Chaos Solitons Fractals 30 (2006) 700–708.
[6] A.T. Ali, E.R. Hassan, General $\exp_a$ function method for nonlinear evolution equations, Appl. Math. Comput. 217 (2010) 451–459.

[7] A. Zafar, The $\exp_a$ function method and the conformable time-fractional KdV equations, Nonlinear Eng. 8 (2019) 728–732.
[8] K. Hosseini, L. Kaur, M. Mirzazadeh, H.M. Baskonus, 1-soliton solutions of the (2+1)-dimensional Heisenberg ferromagnetic spin chain model with the beta time derivative, Opt. Quantum Electron. 53 (2021) 125.
[9] N.A. Kudryashov, One method for finding exact solutions of nonlinear differential equations, Commun. Nonlinear Sci. Numer. Simul. 17 (2012) 2248–2253.
[10] S.M. Ege, E. Misirli, The Modified Kudryashov Method For Solving Some Fractional-Order Nonlinear Equations, Adv. Differ. Equ. 2014 (2014) 135.
[11] D. Kumar, M. Kaplan, Application of the modified Kudryashov method to the generalized Schrödinger–Boussinesq equations, Opt. Quantum Electron. 50 (2018) 329.
[12] K. Hosseini, A. Korkmaz, A. Bekir, F. Samadani, A. Zabihi, M. Topsakal, New wave form solutions of nonlinear conformable time-fractional Zoomeron equation in (2+1)-dimensions, Waves Random Complex Media 31 (2021) 228–238.
[13] N.A. Kudryashov, Method for finding highly dispersive optical solitons of nonlinear differential equation, Optik 206 (2020) 163550.
[14] N.A. Kudryashov, Highly dispersive solitary wave solutions of perturbed nonlinear Schrödinger equations, Appl. Math. Comput. 371 (2020) 124972.
[15] K. Hosseini, M. Mirzazadeh, M. Ilie, S. Radmehr, Dynamics of optical solitons in the perturbed Gerdjikov–Ivanov equation, Optik 206 (2020) 164350.
[16] K. Hosseini, M. Mirzazadeh, S. Salahshour, D. Baleanu, A. Zafar, Specific wave structures of a fifth-order nonlinear water wave equation, J. Ocean Eng. Sci. (2021), doi:10.1016/j.joes.2021.09.019.
[17] E. Yaşar, Variational principles and conservation laws to the Burridge–Knopoff equation, Nonlinear Dyn. 54 (2008) 307–312.
[18] M.S. Hashemi, S. Abbasbrandy, M.S. Alhuthali, H.H. Alsulami, Conservation laws and symmetries of mKdV-KP equation, Rom. J. Phys. 60 (2015) 904–917.
[19] E. Yaşar, On the conservation laws and invariant solutions of the mKdV equation, J. Math. Anal. Appl. 363 (2010) 174–181.
[20] F. Taşcan, A. Yakut, Conservation laws and exact solutions with symmetry reduction of nonlinear reaction diffusion equations, Int. J. Nonlinear Sci. Numer. Simul. 16 (2015) 191–196.
[21] A. Akbulut, F. Taşcan, Application of conservation theorem and modified extended tanh-function method to (1+1)-dimensional nonlinear coupled Klein—Gordon–Zakharov equation, Chaos Solitons Fractals 104 (2017) 33–40.
[22] K. Hosseini, A. Akbulut, D. Baleanu, S. Salahshour, The Sharma–Tasso–Olver–Burgers equation: its conservation laws and kink solitons, Commun. Theor. Phys. 74 (2022) 025001.
[23] N.H. Ibragimov, A new conservation theorem, J. Math. Anal. Appl. 333 (2007) 311–328.
[24] A.H. Arnous, M. Mirzazadeh, A. Akbulut, L. Akinyemi, Optical solutions and conservation laws of the Chen–Lee–Liu equation with Kudryashov's refractive index via two integrable techniques, Waves Random Complex Media, 2022, doi:10.1080/17455030.2022.2045044.
[25] A. Akbulut, H. Rezazadeh, M.S. Hashemi, F. Taşcan, The (3+1)-dimensional Wazwaz–KdV equations: the conservation laws and exact solution, Int. J. Nonlinear Sci. Numer. Simul. (2021), doi:10.1515/ijnsns-2021-0161.
[26] A.M. Wazwaz, *N*-soliton solutions for the combined KdV-CDG equation and the KdV-Lax equation, Appl. Math. Comput. 203 (2008) 402–407.
[27] A. Biswas, G. Ebadi, H. Triki, A. Yildirim, N. Yousefzadeh, Topological soliton and other exact solutions to KdV–Caudrey–Dodd–Gibbon equation, Results Math. 63 (2013) 687–703.
[28] J.M. Tu, S.F. Tian, M.J. Xu, T.T. Zhang, Quasi-periodic waves and solitary waves to a generalized KdV–Caudrey–Dodd–Gibbon equation from fluid dynamics, Taiwan. J. Math. 20 (2016) 823–848.
[29] M.A. Akbar, N.H.M. Ali, T. Tanjim, Adequate soliton solutions to the perturbed Boussinesq equation and the KdV–Caudrey–Dodd–Gibbon equation, J. King Saud Univ. Sci. 32 (2020) 2777–2785.
[30] M.I. Asjad, H. Ur Rehman, Z. Ishfaq, J. Awrejcewicz, A. Akgül, M.B. Riaz, On soliton solutions of perturbed Boussinesq and KdV–Caudrey–Dodd–Gibbon equations, Coatings 11 (2021) 1429.
[31] H. Ma, H. Huang, A. Deng, Soliton molecules, asymmetric solitons and hybrid solutions for KdV-CDG equation, Partial Differ. Equ. Appl. Math. 5 (2022) 100214.
[32] M.S. Bruzón, M.L. Gandarias, M. Torrisi, R. Tracinà, Symmetries and special solutions of a parabolic chemotaxis system, Math. Methods Appl. Sci. 44 (2021) 2050–2058.
[33] A.C.J. Luo, R.K. Gazizov, Symmetries and Applications of Differential Equations, Springer, Singapore, 2021.
[34] N.H. Ibragimov, E.D. Avdonina, Nonlinear self-adjointness, conservation laws, and the construction of solutions of partial differential equations using conservation laws, Russ. Math. Surv. 68 (2013) 889.
[35] L.R. Galiakberova, N.H. Ibragimov, Nonlinear self-adjointness of the Krichever–Novikov equation, Commun. Nonlinear Sci. Numer. Simul. 19 (2014) 361–363.
[36] Y. Zhou, W.X. Ma, Complexiton solutions to soliton equations by the Hirota method, J. Math. Phys. 58 (2017) 101511.
[37] K. Hosseini, M. Mirzazadeh, M. Aligoli, M. Eslami, J.G. Liu, Rational wave solutions to a generalized (2+1)-dimensional Hirota bilinear equation, Math. Model. Nat. Phenom. 15 (2020) 61.
[38] L. Akinyemi, M. Şenol, U. Akpan, H. Rezazadeh, An efficient computational technique for class of generalized Boussinesq shallow-water wave equations, J. Ocean Eng. Sci. (2022), doi:10.1016/j.joes.2022.04.023.
[39] L. Akinyemi, U. Akpan, P. Veeresha, H. Rezazadeh, M. Inc, Computational techniques to study the dynamics of generalized unstable nonlinear Schrödinger equation, J. Ocean Eng. Sci. (2022), doi:10.1016/j.joes.2022.02.011.
[40] L. Akinyemi, M. Inc, M. Khater, H. Rezazadeh, Dynamical behaviour of Chiral nonlinear Schrödinger equation, Opt. Quantum Electron. 54 (2022) 191.
[41] M. Senol, E. Az-Zobi, L. Akinyemi, A. Alleddawi, Novel soliton solutions of the generalized (3+1)-dimensional conformable KP and KP–BBM equations, Comput. Sci. Eng. 1 (2021) 1–29.
[42] Z.U.A. Zafar, L. Akinyemi, H. Rezazadeh, M. Inc, Non-Topological, topological and rogue wave soliton solutions for Sharma Tasso Olver equation, J. Ocean Eng. Sci. (2022), doi:10.1016/j.joes.2022.04.028.
[43] S. Kumar, S. Malik, H. Rezazadeh, L. Akinyemi, The integrable Boussinesq equation and its breather, lump and soliton solutions, Nonlinear Dyn. 107 (2022) 2703–2716.
[44] M. Kaplan, M.N. Ozer, Multiple-soliton solutions and analytical solutions to a nonlinear evolution equation, Opt. Quantum Electron. 50 (2018) 2.
[45] M. Kaplan, M.N. Ozer, Auto-Bäcklund transformations and solitary wave solutions for the nonlinear evolution equation, Opt. Quantum Electron. 50 (2018) 33.
[46] N. Raza, M. Kaplan, A. Javid, M. Inc, Complexiton and resonant multi-solitons of a (4+1)-dimensional Boiti–Leon–Manna–Pempinelli equation, Opt. Quantum Electron. 54 (2022) 95.
[47] M.K.A. Kaabar, M. Kaplan, Z. Siri, New exact soliton solutions of the (3+1)-dimensional conformable Wazwaz–Benjamin–Bona–Mahony equation via two novel techniques, J. Funct. Spaces 2021 (2021) 4659905.
[48] M. Kaplan, Applications of two reliable methods for solving a nonlinear conformable time-fractional equation, Opt. Quantum Electron. 49 (2017) 312.
[49] M. Kaplan, Two different systematic techniques to find analytical solutions of the (2+1)-dimensional Boiti–Leon–Manna–Pempinelli equation, Chin. J. Phys. 56 (2018) 2523–2530.
[50] M. Kaplan, A. Bekir, A novel analytical method for time-fractional differential equations, Optik 127 (2016) 8209–8214.